Coulomb's Law

$$\vec{E}=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r')}{|\vec{r}-\vec{r}'|^3}\left(\vec{r}-\vec{r}'\right)d\tau,\ V=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r')}{|\vec{r}-\vec{r}'|}\ d\tau$$

Electrostatics

$$\vec{\nabla}.\vec{E}=\rho/\varepsilon_0\Rightarrow\oint\vec{E}.\hat{n}\ da=Q_{enclosed}/\varepsilon_0$$

$$\nabla\times\vec{E}=0,$$

$$\vec{E}=-\vec{\nabla}V,\ V=-\int\vec{E}.\ d\vec{l}$$

$$\nabla^2V=-\rho/\varepsilon_0$$

Potential and field due to Dipole

$$\vec{p}=q\vec{d}=\int\vec{r}'\rho(\vec{r}')\ d\tau$$

$$V_{dipole}=\frac{p\cos(\theta)}{4\pi\varepsilon_0 r^2}$$

$$\vec{E}_{dipole}=\frac{p}{4\pi\varepsilon_0 r^3}\left[2\cos(\theta)\hat{r}+\sin(\theta)\hat{\theta}\right]$$

Force and Torque

$$\vec{F}=q\vec{E},\ \vec{F}=\left(\vec{p}.\vec{\nabla}\right)\vec{E},\ \vec{N}=\vec{p}\times\vec{E}$$

Electrostatics in material

$$\vec{D}=\varepsilon_0\vec{E}+\vec{P}$$

$$\vec{\nabla}.\vec{D}=\rho_f\Rightarrow\oint\vec{D}.\hat{n}\ da=\left(Q_f\right)_{enclosed}$$

Bound charge densities

$$\vec{\nabla}.\vec{P}=-\rho_b,\ \vec{P}.\hat{n}=\sigma_b$$

Linear Dielectric

$$\vec{P}=\varepsilon_0\chi_e\vec{E},\ \vec{D}=\varepsilon_0(1+\chi_e)\vec{E}=\varepsilon\vec{E}$$

$$\varepsilon=\varepsilon_0(1+\chi_e)$$

Frozen Polarization: Nonlinear Dielectric

Capacitance

$$C=Q/V$$

Energy

$$W_e=\frac{\varepsilon_0}{2}\int E^2d\tau=\frac{1}{2}\int\rho V\ d\tau=\sum\frac{\rho_i V_i}{2}$$

Energy Stored in Dielectric

$$W_e=\frac{1}{2}\int\vec{E}.\vec{D}\ d\tau$$

Conductor with surface charge: Force/area

$$\vec{f}=\frac{\sigma^2\hat{n}}{2\varepsilon_0}$$

Biot-Savart law

$$\vec{B}=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(r')\times\left(\vec{r}-\vec{r}'\right)}{|\vec{r}-\vec{r}'|^3}d\tau,\ \vec{A}=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(r')}{|\vec{r}-\vec{r}'|}\ d\tau$$

Magnetostatics

$$\vec{\nabla}.\vec{B}=0$$

$$\vec{\nabla}\times\vec{B}=\mu_0\vec{J}\Rightarrow\oint\vec{B}.\vec{dl}=\mu_0 I_{enclosed}$$

$$\vec{B}=\vec{\nabla}\times\vec{A},$$

$$\nabla^2\vec{A}=-\mu_0\vec{J}$$

Potential and field due to Dipole

$$\vec{m}=I\ \int d\vec{a}=\frac{1}{2}\int\vec{r}'\times\vec{J}(\vec{r}')\ d\tau$$

$$\vec{A}_{dipole}=\frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}=\frac{\mu_0}{4\pi}\frac{m\sin(\theta)}{r^2}\hat{\phi}$$

$$\vec{B}_{dipole}=\frac{\mu_0}{4\pi}\frac{m}{r^3}\left[2\cos(\theta)\hat{r}+\sin(\theta)\hat{\theta}\right]$$

Force and Torque

$$\vec{F}=q\left(\vec{v}\times\vec{B}\right),\ \vec{F}=\vec{\nabla}\left(\vec{m}.\vec{B}\right),\ \vec{N}=\vec{m}\times\vec{B}$$

Magnetostatics in a material

$$\vec{H}=\left(\vec{B}/\mu_0\right)-\vec{M},\ \vec{B}=\mu_0\left(\vec{H}+\vec{M}\right)$$

$$\vec{\nabla}\times\vec{H}=\vec{J}_f\Rightarrow\oint\vec{H}.\vec{dl}=\left(I_f\right)_{enclosed}$$

Bound current densities

$$\vec{\nabla}\times\vec{M}=\vec{J}_b,\ \vec{M}\times\hat{n}=\vec{K}_b$$

Linear magnetic material

$$\vec{M}=\chi_m\vec{H},\ \vec{B}=\mu_0(1+\chi_m)\vec{H}=\mu\vec{H}$$

$$\mu=\mu_0(1+\chi_m)$$

χ_m negative for Diamagnetic,

χ_m positive for paramagnetic

Frozen in magnetization: Nonlinear

See Ferromagnetic material and Hysteresis Loop

Boundary Conditions:

$$V_2-V_1=0,\ \vec{A}_2-\vec{A}_1=0$$

$$\hat{n}.(\vec{D}_2-\vec{D}_1)=\sigma_f,\ \hat{n}.(\vec{B}_2-\vec{B}_1)=0$$

$$\hat{n}\times(\vec{E}_2-\vec{E}_1)=0,\ \hat{n}\times(\vec{H}_2-\vec{H}_1)=\vec{K}_f$$

$\hat{n}$ is unit vector from medium 1 to 2.

Jackson: ***B*** -> magnetic induction and ***H*** -> magnetic field