Radiation

First find vector potential for different multipole radiations and then calculate B field and then calculate E field using following relations:

$$\vec{B}_{rad}(\vec{r},t) \approx -\frac{\hat{r}}{c} \times \dot{\vec{A}}_{rad}(\vec{r},t),$$

$$\vec{E}_{rad}(\vec{r},t) \approx c\left[\vec{B}_{rad}(\vec{r},t) \times \hat{r}\right] \approx \left[\hat{r} \times \left(\hat{r} \times \dot{\vec{A}}_{rad}(\vec{r},t)\right)\right]$$

$$\frac{dP}{d\Omega} = \frac{1}{2\mu_0}\mathrm{Re}\left[r^2 \hat{r} \bullet (\vec{E} \times \vec{B}^*)\right] = \frac{c}{2\mu_0}\left[r^2 \left|\vec{B}\right|^2\right] = \frac{1}{2\mu_0 c}\left[r^2 \left|\vec{E}\right|^2\right],$$

$$P_{total} = \int \frac{dP}{d\Omega} d\Omega,$$

Electric Dipole Radiation:

$$\vec{A}_d(\vec{r},t) \approx \frac{\mu_0}{4\pi}\frac{\dot{\vec{p}}(t_r)}{r},\ \vec{B}_d(\vec{r},t) \approx -\frac{\mu_0}{4\pi c}\frac{\hat{r} \times \ddot{\vec{p}}(t_r)}{r},\ \vec{E}_d \approx \frac{\mu_0}{4\pi}\frac{\hat{r} \times \left(\hat{r} \times \ddot{\vec{p}}(t_r)\right)}{r},$$

$$\vec{p}(t_r) = \int \vec{r}\,'\ \rho(r',t_r)\ d^3r',$$

$$\frac{dP_d}{d\Omega} \approx \frac{c}{2\mu_0}\left[r^2 \left|\vec{B}_d\right|^2\right] \approx \frac{\mu_0}{32\pi^2 c}\left[\left|\hat{r} \times \ddot{\vec{p}}(t_r)\right|^2\right],\ P_d \approx \frac{\mu_0 \omega^4 p_0^2}{12\pi\ c}.$$

Magnetic Dipole Radiation:

$$\vec{A}_m(\vec{r},t) = \frac{\mu_0}{4\pi c}\frac{\dot{\vec{m}}(t_r) \times \hat{r}}{r},\ \vec{B}_m(\vec{r},t) \approx \frac{\mu_0}{4\pi c^2}\frac{\hat{r} \times \left(\hat{r} \times \ddot{\vec{m}}(t_r)\right)}{r},\ \vec{E}_m \approx \frac{\mu_0}{4\pi c}\frac{\left(\hat{r} \times \ddot{\vec{m}}(t_r)\right)}{r},$$

$$\vec{m}(t_r) = \frac{1}{2}\int \left(\vec{r}\,' \times \vec{J}(r',t_r)\right)\ d^3r',$$

$$\frac{dP_m}{d\Omega} \approx \frac{1}{2\mu_0 c}\left[r^2 \left|\vec{E}_m\right|^2\right] \approx \frac{\mu_0}{32\pi^2 c^3}\left[\left|\hat{r} \times \ddot{\vec{m}}(t_r)\right|^2\right],\ P_m \approx \frac{\mu_0 \omega^4 m_0^2}{12\pi\ c^3}.$$

Electric Quadrupole Radiation:

$$\vec{A}_Q(\vec{r},t) = \frac{\mu_0}{24\pi c}\frac{\ddot{\vec{Q}}(t_r)}{r},\ \vec{B}_Q(\vec{r},t) \approx -\frac{\mu_0}{24\pi c^2}\frac{\hat{r} \times \dddot{\vec{Q}}(t_r)}{r},\ \vec{E}_Q \approx \frac{\mu_0}{24\pi c}\frac{\hat{r} \times \left(\hat{r} \times \dddot{\vec{Q}}(t_r)\right)}{r},$$

$$\frac{dP_Q}{d\Omega} \approx \frac{c}{2\mu_0}\left[r^2 \left|\vec{B}_Q\right|^2\right] \approx \frac{\mu_0}{2(24\pi)^2 c^3}\left[\left|\hat{r} \times \dddot{\vec{Q}}(t_r)\right|^2\right],\ P_Q \approx \frac{\mu_0 \omega^6 Q_0^2}{240 \times 4\pi\ c^3}.$$

where

$$\vec{Q} = \hat{e}_1\left[Q_{11}n_1 + Q_{12}n_2 + Q_{13}n_3\right] + \hat{e}_2\left[Q_{21}n_1 + Q_{22}n_2 + Q_{23}n_3\right] + \hat{e}_3\left[Q_{31}n_1 + Q_{32}n_2 + Q_{33}n_3\right]$$

and

$$Q_{ij}(t_r) = \int \left[3x'_i\, x'_j - r'^2\, \delta_{ij}\right]\rho(r',t_r)\ d^3r',$$