Laser Physics: PHYS 5734/4734

Spring 2009, Homework Set - 3

Due: Thursday, Feb. 26, 2009.

11 A collimated gaussian light beam is incident on a lens of focal length f. If geometrical optics rules applied exactly, the light would be focused to a point at the focal point of the lens. Use gaussian beam results to find the location of the beam waist after the lens and spot size there. Find the focus shift (distance of the waist from the focal point of the lens) in terms of given parameters. Compute its value for $\lambda = 500$ nm, $w_o = 1$ mm and (i) $f = 50$ mm (ii) $f = 5$ m.

12(a) If we take the fundamental gaussian solution as giving the value of, say, the $x-$component $\mathcal{E}_x$ of the electric field in a gaussian beam propagating in z-direction, show that $\mathcal{E}_z$ cannot be zero. Find $\mathcal{E}_z$. Similarly, show that the magnetic field has $\mathcal{B}_y$ and $\mathcal{B}_z$ components. Find these components for the fundamental gaussian beam.

(b)* Compute the time averaged Poynting vector $\boldsymbol{S}_{\rm av} = \frac{1}{2\mu_0}\left[\boldsymbol{\mathcal{E}}^* \times \boldsymbol{\mathcal{B}}\right]$ and show that it correctly describes the focusing and defocusing of a gaussian beam.

13(a) Given the round trip matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ for an optical cavity starting from some reference plane, show that the complex beam parameter q of the trapped gaussian beam in that reference plane is given by

$$q = \frac{A-D}{2C} \pm i\frac{\sqrt{4-(A+D)^2}}{2C}\,.$$

Use the $ABCD$ matrix derived in class for a two mirror cavity with the reference plane immediately in front of mirror 1 and check that this formula leads to the results derived in class for the spot size and the phase front radius of curvature.

(b) If the reference plane is a waist of the trapped gaussian beam, determine the conditions on the matrix elements and the spot size at the waist in terms of the matrix elements.

14(a) Consider a near-planar symmetric resonator made with mirrors of radius R and separation $L \ll R$. Obtain an approximate expression for the spot sizes on the mirrors and at the waist. Use your results to calculate the spot sizes when $R = 8$ m and $L = 1$ m at a wavelength of 514 nm.

(b) Consider a near concentric resonator composed of two mirrors of radius R and separation $L = 2R-\delta$. Give an approximate expression for the spot sizes on the mirrors and at the waist. Compare this to the result of part (a) and comment on the similarity or difference of the two results.

15(a) Find the waist location, minimum spot size, and spot sizes at the mirrors for a laser resonator formed by two mirrors with radii of curvature $R_1 = 20$ cm and $R_2 = 32$ cm separated by $L = 16$ cm. Take the wavelength of operation to be 1.06 μm.

(b) Find the axial and transverse mode separation.