Laser Physics: PHYS 5734/4734

Spring 2009, Homework Set - 4

Due: Friday, March 6, 2009.

16 We derived an expression for the allowed frequencies of a two mirror cavity in class. Now suppose you have a ring cavity formed by, say, arranging three mirrors at the vertices of a triangle. Assume that the cavity is stable and has a perimeter L. Find an expression for the allowed frequencies $\nu_{p\ell m}$ and axial mode spacing $\Delta\nu_{ax}$. For $L = 50$ cm and $n = 1$ compare these with the corresponding values for a two-mirror cavity.

17 Fabry-Perot interferometer (FPI) as an optical spectrum analyzer: A Fabry-Perot interferometer can be used to analyze the spectral components of a laser beam as described in section 19.3 of Siegman or see the write up for the fourth lab. The peak transmission wavelength (or frequency) of the FPI is scanned by moving one of the mirrors. Recording the optical power transmitted through the FPI as a function of the cavity length then allows measurement of the spectral content of the light. There are competing constraints on the design of the interferometer. For example, we saw in class that there are actually a series of equally spaced (in frequency) transmission peaks in a FPI. If the separation of adjacent transmission peaks is narrower than the spectrum of the light, an ambiguity arises in uniquely associating the power transmitted by the FPI with a particular frequency (the origin of the term "free spectral range" for the axial mode separation $\Delta\nu_{ax} = c/2L$). This constraint favors a shorter cavity. On the other hand, we also saw in class that for a given mirror reflectivity (and hence given finesse) the width of the transmission peaks $\Delta\nu_c$ (and hence the minimum resolvable frequency difference) is proportional to $\Delta\nu_{ax}$, so that shortening the cavity (to increase the free spectral range) brings with it poorer frequency resolution.

(a) Explain how the "finesse" discussed in class can reasonably be interpreted as the number of distinct frequency components that can be resolved unambiguously by the FPI. So far our discussion has been appropriate to a plane parallel mirror FPI. Now, we discussed in class that the marginal stability of planar interferometers leads to difficulty in practical operation of such devices, especially in designs with high finesse. To avoid this problem, most real FPI use curved mirrors. However, the existence of transverse modes with different resonance frequencies [Fig 19-16 Siegman or Figure 3.3 of Lecture Notes] introduces another complication, such that one again becomes unable to associate the transmitted power with a unique frequency unless one manages to excite only a single transverse mode of the cavity, which is generally difficult.

(b) Explain why the confocal design is better than the other curved mirror cavities as a spectral analysis tool. Also, how does its effective free spectral range (defined as the frequency separation of closest modes) compare to that of a planar FPI?

(c) Now suppose the device is 20 cm long and is designed to measure red light. If the finesse of the device is 100, how close must g be to 0, and thus how close must the mirror curvature be to 20 cm, in order that the resonance frequency of none of the modes $\nu_{p,\ell m}$ $(\ell, m \leq 2)$ fall outside the full-width at half maximum of the TEM_{00} mode (and hence lower the effective resolution of the device)?

18 **Parameters of a Fabry-Perot Interferometer:** A Fabry-Perot consists of two identical air-spaced mirrors. It is illuminated by monochromatic light wave of tunable

frequency. From a measurement of the trasnmitted intensity versus the frequency of the input wave we find that the free spectral range of the interferometer is 3.00 GHz, its resolution is 60 MHz, and the peak trasnmission is 50%. Calculate the spacing, finesse, mirror reflectivity, and the internal loss of the interferometer.

19 We have derived the expressions for the location of the beam waist relative to the mirrors (z_1 and z_2). Thus the waist is located a distance $|z_1|$ to the left of mirror M_1 or a distance $|z_2|$ to the right of M_2. However, as the beam exits the cavity, it suffers refraction at the exit mirror so that the apparent waist position for the emergent beam is different from that given by z_1 or z_2. Assume for concreteness that we have an approximately confocal cavity with mirror separation d. How far is the apparent waist from the exit mirror? What is the shift? (Treat the output mirror as a thin plano-concave lens with refractive index n). Assume $n = 1.5$ and determine the actual shift in terms of d.

20 **Cavity ringing:** One way to measure small absorption is to place the absorber in a high-Q resonator and shine a short pulse of laser light on to the resonator. As the pulse bounces around the cavity and each time the pulse hits the output mirror, it produces a transmitted pulse. From the decay of the pulse amplitude we can determine cavity losses. Consider a resonator consisting of two identical mirrors separated by a distance L in air and suppose that it is illuminated by a 1-ps pulse from an external source at wavelength 600 nm. The output consists of a regular sequence of 1-ps pulses separated by 10 ns. The energy of the pulses decreases exponentially with time with a time constant of 100 ns. Calculate the cavity length and mirror reflectivity.