Laser Physics: PHYS 5734
Spring 2009, Homework Set - 7

Due: Friday, April 30

30. **Frequency dependence of laser power :** Many applications require a tunable single-frequency laser beam. To see how this can be achieved, consider a laser with a homogeneously broadened gain profile operating at a fixed pump rate above threshold. To ensure single-mode operation one can insert a narrow bandpass filter in the laser cavity, whose center frequency can be tuned across the gain profile, allowing oscillation only at the filter frequency.

(a) Determine the output power versus frequency curve for such a laser. You can assume small gain and small output coupling approximation holds. Express your result in terms the saturation intensity I_s and laser pump ratio r at line center and homogeneous linewidth.

(b) What will be the frequency tuning range for this laser? The situation modeled here is an idealization of that encountered in CW tunable systems like dye lasers.

31. **Relaxation oscillations** Investigate the possibility of relaxation oscillations in the following systems and find their life-time, frequency and period.

(a) A Nd:YAG laser (λ =1.064 μm) operating 50% above threshold has a cavity 50 cm long and total loss per pass of $\mathcal{L}$=0.12. The lifetime of the upper level is 0.23 ms and the lifetime of the lower level is much shorter than this. Determine the period and the decay time for relaxation oscillations.

(b) A dye laser is operating with a pump ratio $r = 2.5$. The cavity is 5 cm long and round trip losses are 5%. Consider the dye-laser to be a four-level system with an upper level life time ($1/\gamma_2$) of 1 ns.

32. **Q-switching in Nd:YAG** ($r = 2.0$)

A Nd:YAG laser operating at wavelength $\lambda = 1.064\,\mu$m consists of an 18-cm long standing wave cavity containing a 5.0 cm long Nd:YAG rod (refractive index $n = 1.8$, upper level lifetime 230 μs) with an effective cross-sectional area of 0.20 cm^2. The laser is operated in the Q-switched regime. Threshold inversion density for the laser is $\Delta\mathcal{N}_{th} = 4.35 \times 10^{17}$ cm^{-3} and it is pumped 2.0 times above threshold. Given that $\mathcal{R}_1 \approx 1$, $\mathcal{R}_2 = 30\%$ and round-trip internal loss $\mathcal{L}_i = 33\%$.

(a) Calculate the total energy extracted from the inversion? What fraction is this of the energy initially stored in the inversion? What is the total pulse energy output?

(b) Calculate the peak photon number in the cavity? What is the peak power output from the laser? What is the pulse width?

33. **Mode-locking in a He:Ne laser :** Consider a 100-cm-long He-Ne laser, with a Doppler linewidth (FWHM) of 1.5 GHz, and Brewster windows that ensure oscillation in single polarization. Take the laser to be pumped so that the unsaturated line center gain is 4 times the threshold value.

(a) How many TEM_{00} modes could oscillate simultaneously?

(b) If all the modes were locked together, what would the repetition rate of the pulses be? Estimate the pulse width.

34. **Mode-locking in a Nd:YAG laser :** A Nd:YAG laser at $\lambda = 1.06\ \mu$m has a homogeneously broadened line whose width (FWHM) is $\Delta\nu_{_H} = 195$ GHz. Assuming a cavity length of $L = 1.5$ m, calculate the expected pulsewidth when the laser is mode-locked by an acousto-optic modulator. Assume the oscillation bandwidth to be equal the gain linewidth. What frequency would be needed to drive the AOM for mode-locking? What would be the expected pulsewidth for a laser with an inhomogeneously broadened gain profile of the same oscillation bandwidth?

35* **Laser turn-on : Bonus problem**

We have seen that when spontaneous emission is included, laser rate equations equations become[1]

$$\frac{dN}{dt} = r_p - KNq - \gamma_2, , \tag{E1}$$

$$\frac{dq}{dt} = Kn(q+1) - \gamma_c q\,. \tag{E2}$$

Show that the cavity photon number q in a single-mode laser near threshold $[|r-1| \ll 1$ so that $Kq/\gamma_2 \ll 1]$ builds up according to

$$\frac{dq}{dt} = \frac{K\,r_p}{\gamma_2}\left[1 - \left(\frac{K}{\gamma_2}\right)q\right](q+1) - \gamma_c q, .$$

In terms of dimensionless variables $x = Kq/\gamma_2 \equiv q/q_s$, $\tau = \gamma_c t$, and pump parameter $r = K\,r_p/\gamma_2\gamma_c = N^{(0)}/N_{th}$ we can rewrite this equation as

$$\frac{dx}{d\tau} = r\,(1-x)\left[x + \frac{1}{q_s}\right] - x\,.$$

(a) Solve this equation in the steady-state and identify the physically acceptable solution.

[1] Note that Eq. (E1) is different from that given in the notes. Note that spontaneous emission into cavity mode is already included in the γ_2-term in this equation for atomic inversion. So in the notes, when we replaced q by $q+1$ in the inversion equation, we should have reduced γ_2 to reflect this otherwise we are counting spontaneous emission into the cavity mode twice. However, for most practical lasers, this change is small and can be ignored. The advantage of replacing q by $q+1$ in both the photon number and inversion equations is that it simplifies the calculation.

(b) Show that with the initial condition $x(0) = 0$, the time dependent solution is given by

$$q(t) = \frac{q_{ss}(1 - e^{-\gamma_c' t})}{1 + \frac{q_{ss}^2}{q_s}\, e^{-\gamma_c' t}} \,,$$

with
$$q_{ss} = \frac{q_s}{2r}\left[(r - 1 - r/q_s) + \sqrt{(r - 1 - r/q_s)^2 + 4r^2/q_s}\,\right],$$

and
$$\gamma_c' = \gamma_c\sqrt{(r - 1 - r/q_s)^2 + 4r^2/q_s}\,.$$

Check that this solution has the correct limits as $t \to 0$ and $t \to \infty$. Plot this solution for $r = 0.98, 1.005$, and 1.02. You may find it convenient to plot $x = q/q_s$ as a function of $\gamma_c t$. Also, depending or r, you may have to go out to fairly large values of $\gamma_c t$ (many cavity life times) before you see the approach to the steady-state. Use a 'reasonable' definition of buildup time and derive an expression for the buildup time of the laser.